Optional Tools of the Wi-Fi protocol: Study in Saturation

A. Lyakhov, V. Vishnevsky

Institute for Information Transmission Problems of RAS
B. Karetny 19, Moscow, 101447, Russia
{vishn, lyakhov}@iitp.ru / WWW: http://www.iitp.ru

Abstract. IEEE 802.11 protocol called also the Wireless Fidelity (Wi-Fi) one specifies a technology for wireless local area networks (LANs) and mobile networking. In contrary to previous works dealing with the Basic Mechanism of the protocol, in this paper we focus on such optional tools as the RTS/CTS technique and packet fragmentation, which are oriented on fight with collisions and noise-induced distortion. With the study, we present an analytical method of estimating both the saturation throughput of a Wi-Fi LAN and probability of a packet rejection occurring when the number of packet transmission retries attains its limit. This method allows finding areas where the optional tools show their efficiency and tuning optimally the corresponding Wi-Fi managed parameters.

1 Introduction

Wireless networks have become an emerging technology for today's communication industry, and this situation is referred as a wireless revolution [10] in network industry. The IEEE 802.11 [1] is a well-consolidated standard for wireless LANs, and several computer and telecommunication industries have launched into the market their IEEE 802.11 products. This standard is permanently developed to provide higher and higher transmission rates: the maximal rate has increased from 2 Mbps in the classical 802.11 protocol appeared in 1997 to 54 Mbps in the 802.11a protocol. As Leonard Kleinrock indicated in [10], 802.11

networks called also the Wi-Fi (Wireless Fidelity) networks represent actively defining best-practices technology for use in industry.

The fundamental access mechanism in the IEEE 802.11 protocol is the Distributed Coordination Function (DCF), which implements the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) method. In early works, performance of the DCF was evaluated either by simulation (e.g., [4]) or by approximate analytical models [5, 6] based on assumptions simplifying considerably the backoff rule. The DCF scheme was studied in depth in [7, 8], and [6], in which analytical methods were developed for evaluating the performance of 802.11 wireless LANs in the saturation conditions when there are always queues for transmitting at every wireless LAN station. This performance index called the *saturation throughput* in [7] was evaluated in the assumption of ideal channel conditions, i.e., in the absence of noise and hidden stations. The assumption of the absence of hidden stations is admissible as a result of a small distance between LAN stations. But if noise is neglected, the throughput may be overestimated, because electromagnetic noise in large cities is inevitable and worsens the throughput due to data distortion. In [7], we have developed the methods of [7, 8], and [6] to study the influence of noise on the Wi-Fi LAN performance.

In this paper, we keep investigating this problem, extending the approach [7] to study such Wi-Fi optional tools as the RTS/CTS technique oriented on fight with collisions and packet fragmentation that allows reducing significantly the influence of noise.

Further in Section 2 we briefly review the DCF operation in saturation and noise. In Sections 3 and 4 we study a fragmented packet transmission process and develop a new analytical method of estimating the saturation throughput and a probability of a packet rejection occurring when the number of packet transmission retries attains its limit. Finally, the obtained results are summarized in section 5.

2 DCF in Saturation

Now we briefly outline the DCF scheme, considering only the aspects that are exhibited in saturation and with absence of hidden stations. This scheme is described in detail in [1].

Under the DCF, data packets are transferred in general via two methods. Short packets of length not greater than $\overline{P}$ are transferred by the Basic Access mechanism. In this mechanism shown in Figure 1, a station confirms the successful reception of a DATA frame by a positive acknowledgment ACK after a short SIFS interval.

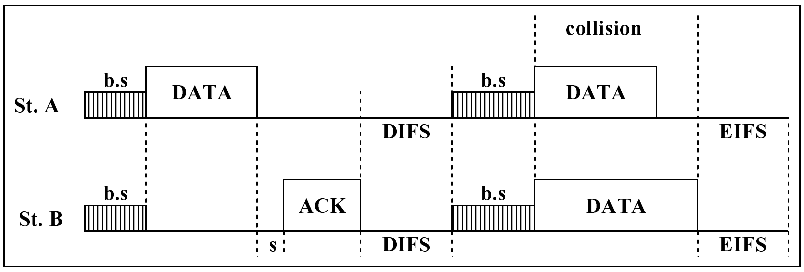

Fig. 1. Basic Access Mechanism (s - SIFS, b.s - backoff slots)

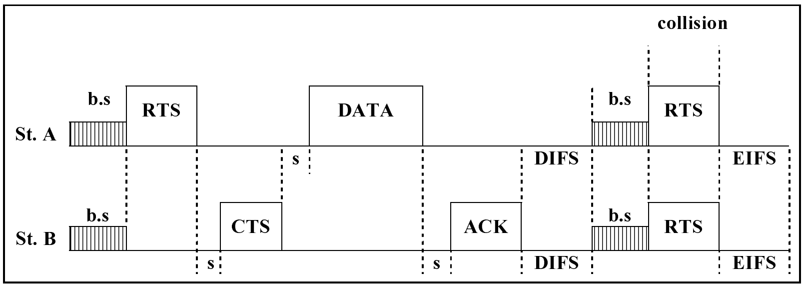

Fig. 2. RTS/CTS mechanism

Packets of length greater than the limit $\overline{P}$ called the RTS threshold in [1] are transferred via the Request-To-Send/Clear-To-Send (RTS/CTS) mechanism. In this case shown in Figure 2, first an inquiring RTS frame is sent to the receiver station, which replies by a CTS frame after a SIFS. Then only a DATA frame is transmitted and its successful reception is confirmed by an ACK frame. Since there are no hidden stations in the considered LAN, all other stations hear the RTS frame transmission and defer from their own attempts. This protects CTS, DATA and ACK frames from a collision-induced distortion. The RTS threshold

$\overline{P}$ is chosen as a result of a reasonable trade-off between the RTS/CTS mechanism overhead consisting in transmitting two additional control frames (RTS and CTS) and reduction of collision duration. Figures 1 and 2 show that the collision duration is determined by the length of the longest packet involved in collision for the Basic Access mechanism, whereas in the RTS/CTS mechanism it is equal to the time of transferring a short RTS frame.

After a packet transfer attempt, the station passes to the backoff state after a DIFS interval if the attempt was successful (i.e., there was no collision, all frames of a packet were transferred without noise-induced distortions) or after an EIFS interval if the attempt failed. The backoff counter is reset to the initial value b, which is called the backoff time, measured in units of backoff slots of duration σ, and chosen uniformly from a set $(0, \ldots, w-1)$. The value w, called the contention window, depends on the number n_r of attempts performed for transmitting the current packet: $w = W_{n_r}$, where

$$W_{n_r} = \begin{cases} W_0 2^{n_r} & \text{for } n_r \leq m \\ W_m & \text{for } n_r > m, \end{cases} \tag{1}$$

i.e., w is equal to the minimum W_0 before the first attempt, then w is doubled after every failed attempt of the current packet transmission, reaching the maximum $W_m = W_0 2^m$. Note that every attempt of transmitting a packet can include transfers of several frames (RTS, CTS, DATA, and ACK). Backoff interval is reckoned only as long as the channel is free: the backoff counter is decreased by one only if the channel was free in the whole previous slot. Counting the backoff slots stops when the channel becomes busy, and backoff time counters of all stations can decrement next time only when the channel is sensed idle for the duration of σ+DIFS or σ+EIFS if the last sensed transmission is successful or failed, respectively. When the backoff counter attains its zero value, the station starts transmission.

In the course of transmission of a packet, the transmitting station counts the number of short (n_s) and long (n_d) retries. Let a source station transfer a DATA frame with a packet of length equal to or less than $\overline{P}$, or an RTS frame. (Retries for these frames are called the short ones in [1].) If a correct ACK or CTS frame is received within timeout, then the n_s-counter is zeroed; otherwise n_s is advanced by one. Similarly, the n_d-counter is zeroed or advanced by one in case of reception or absence of a correct ACK frame (within timeout) confirming the successful transfer of a DATA frame with a packet of length greater than $\overline{P}$ (transfer retries for that sort of DATA frames are called the long retries in [1]). When any of these counters n_s and n_d attains its limit N_s or N_d, respectively, the current packet is rejected. After the rejection or success of a packet transmission, the next packet is chosen (due to saturation) with zeroing the values of n_r, n_s, and n_d.

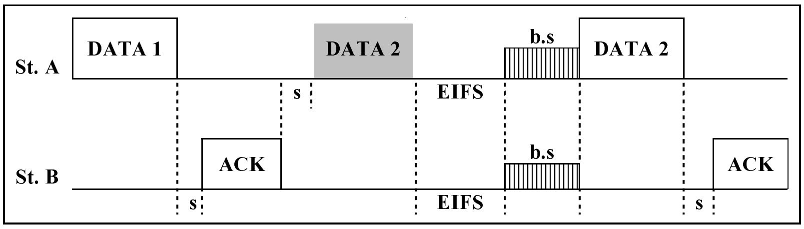

Fig. 3. Fragmented packet transmission

For reducing the influence of noise, Standard 802.11 [1] recommends subdividing a packet longer than a fragmentation threshold L_f into fragments of size L_f (except for the last fragment). Thus, a packet is transferred as a continuous chain of DATA frames, which contain sequential fragments and are interspaced with ACK frames and short interframe SIFS intervals (see Figure 3). If a fragment is distorted (fragment 2 in Figure 3), the station passes to the backoff state, advancing the retry counters n_r and n_d by one, and thereafter the packet transmission is resumed precisely with this distorted fragment. Thus, the transmission of a packet can be considered as a transfer of one or several continuous chains of frames (there are two chains in Figure 3), and these chains are separated by backoff intervals. A DATA frame being the first in a chain and containing a fragment of length greater than the threshold $\overline{P}$ is transferred upon receipt of a CTS frame in response to an inquiring RTS frame. Subsequent DATA frames cannot be involved into collisions because all other stations hear the transmission of previous DATA and ACK frames and defer from their attempts. Notice that, in contrary to the n_r-counter referring to a whole packet, counters n_s and n_d refer to a fragment and are zeroed after the fragment transmission success.

As in [7, 8], and [7], to study the DCF, we adopt the following assumption: all stations change their backoff counters after a DIFS or an EIFS interval closing a transmission attempt, i.e., the source station (or stations in case of collision), which has performed a transmission, modifies its contention window w and chooses randomly the backoff counter value from the set $(0, \ldots, w-1)$, while other stations just decrease their backoff counters by 1. (In reality [1], other stations can do it only after a backoff slot σ since the end of the DIFS or EIFS interval.) Thus, at the beginning of each slot any station can start its transmission. As shown in [6], this assumption does not affect significantly the throughput estimation results with the W_0 values recommended in [1].

3 Throughput Evaluation

Let us consider a wireless LAN of N statistically homogeneous stations working in saturation. In fact, we mean by N not a number of all stations of the LAN, but a number of active stations, whose queues are not empty for a quite long observation interval. By statistically homogeneity of stations, we mean the following: (i) the lengths of packets chosen by every station from the queue have identical probability distribution $\{d_\ell, \quad \ell = 1, \ldots, \ell_{\max}\}$; (ii) all stations adopt the same RTS and fragmentation thresholds ($\overline{P}$ and L_f); (iii) the propagation delay is assumed the same for all pairs of stations and equal to a small value δ. Since the distance between stations is small, we assume that there are no hidden stations and noise occurs concurrently at all stations. The last assumption implies that all stations "sense" the common wireless channel identically.

As in [7] and [7], let us subdivide the time of the LAN operation into non-uniform virtual slots such that every station changes its backoff counter at the start of a virtual slot and can begin transmission if the value of the counter becomes zero. Such a virtual slot is either (a) an "empty" slot in which no station transmits, or (b) a "successful" slot in which one and only one station transmits, or (c) a "collisional" slot in which two or more stations transmit.

As in [7, 8] and [7], we assume that the probability that a station starts transmitting a packet in a given slot depends neither on the previous history, nor on the behavior of other stations, and is equal to τ, which is the same for all stations. Hence the probabilities that an arbitrarily chosen virtual slot is "empty" (p_e), "successful" (p_s), or "collisional" (p_c) are

$$p_e = (1-\tau)^N, \quad p_s = N\tau(1-\tau)^{N-1}, \quad p_c = 1 - p_e - p_s. \tag{2}$$

With every packet transmission attempt, a chain of data frames is tried to be transferred, the first frame of the chain containing the first fragment which has not been transferred correctly yet. So let us associate every packet transmission attempt with a pair (ℓ, k) where ℓ is the length (in bytes) of the packet which the chain is related to and $k+1$ is the number of the packet fragments remaining to be transferred. Here $k = 0, \dots, K(\ell)$, where $K(\ell)$ is the integer part of the ratio $(\ell - 1)/L_f$. Let $r^1_{\ell k}$ and r^0_ℓ denote the lengths of the first and last fragments of the packet remainder transferred in the current attempt. For example, let us consider an attempt of transmitting a packet of length $l = 4.5L_f$, assuming that two first fragments of this packet have already been transferred successfully and the data frame containing the third fragment has been distorted with the previous attempt. Then the chain includes data frames with two L_f-length fragments and the last fragment of length of $L_f/2$, which form the packet remainder, and this attempt is associated with the pair $(4.5L_f, 2)$; moreover, $r^1_{\ell k} = L_f$ and $r^0_\ell = L_f/2$. Note that transmitting the first data frame of a chain described by (ℓ, k) is anticipated by an RTS-CTS sequence if $r^1_{\ell k} > \overline{P}$.

Let $\widehat{d}_{\ell k}$ be the probability that an arbitrarily chosen attempt of a packet transmission is related with a concrete pair (ℓ, k).

The throughput S is defined as the average number of successfully transferred data bits per a second. Note that we should count these data bits only after a successful completion of transmitting a whole packet, but not after each fragment transmission, because a packet transmission process can end with the packet rejection in spite of some fragments of the packet can be transferred successfully. Thus, the throughput is determined by the formula

$$S = p_s \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \sum_{k=0}^{K(\ell)} 8\ell \pi_h(\ell, k) \widehat{d}_{\ell k} / [p_e \sigma + p_s T_s + p_c T_c], \tag{3}$$

where T_s and T_c are the mean duration of "successful" and "collisional" slots, respectively, that is, the denominator is the average duration of a virtual slot. In the numerator of (5), $\pi_h(\ell, k)$ is the probability that an attempt associated with a pair (ℓ, k) and carried out in a "successful" slot completes successfully a transmission of the whole packet of length ℓ.

The duration of a "collisional" slot is the sum of time of transmitting the longest frame involved in collision and an EIFS interval. Disregarding the probability of collision of three or more frames, we obtain the formula for the mean duration of a "collisional" slot

$$T_c = \sum_{r=1}^{\min(\overline{P},L_f)} t_d(r)(d_r^*)^2 + 2 \sum_{r=1}^{\min(\overline{P},L_f)} \sum_{k=r_{\min}}^{r-1} t_d(r) d_r^* d_k^*$$

$$+\mathbf{1}(L_f > \overline{P})\, 2 \sum_{r=1}^{\min(\overline{P},L_f)} \sum_{k=\overline{P}+1}^{L_f} t_d(r) d_r^* d_k^*$$

$$+\mathbf{1}(L_f > \overline{P}) t_{RTS} \left(\sum_{r=\overline{P}+1}^{L_f} d_r^* \right)^2 + EIFS + \delta, \qquad (4)$$

where $t_d(r) = H + 8r/V$ is the transmission time of a DATA frame including a fragment of length r and a header transmitted in time H, V is the channel rate (in bits per a second), and t_{RTS} is the transfer time for an RTS frame (according to [1], $t_{RTS} < H$). Moreover, d_r^* is the probability that a chain with first fragment of length r is transferred in the current attempt, i.e.,

$$d_r^* = \varphi(r) = \sum_{k=0}^{k_{\max}(r)} \widehat{d}_{kL_f+r,\,0}, \quad r < L_f, \quad d_{L_f}^* = 1 - \sum_{\ell=1}^{\ell_{\max}} \widehat{d}_{\ell,0} + \varphi(L_f),$$

where $k_{\max}(r)$ is the integer part of the ratio $(\ell_{\max} - r)/L_f$. Here and in what follows, we use the Boolean function $\mathbf{1}$(condition) which takes the value 1 if the condition within brackets holds.

Now we study a "successful" slot. At the beginning of this slot, only one station makes an attempt of transmission which with probability $\widehat{d}_{\ell k}$ is related with the pair (ℓ, k). This attempt is concluded successfully for the pair (ℓ, k), i.e., with successful transfer of a whole packet of length ℓ, with probability $\pi_h(\ell, k)$ if none of the frames exchanged between the sender and receiver in this process is distorted by noise, that is,

$$\pi_h(\ell, k) = Q(r_{\ell k}^1)[1 - \xi(L_f)]^k [1 - \xi(r_\ell^0)], \qquad (5)$$

where

$$Q(\ell_f) = 1 - \xi_{rc} = (1 - \xi_r)(1 - \xi_a)$$

for $\ell_f > \overline{P}$ and $Q(\ell_f) = 1$ for $\ell_f \leq \overline{P}$,

$$\xi(l_f) = 1 - [1 - \xi_d(l_f)](1 - \xi_a),$$

at last, $\xi_d(\ell_f)$, ξ_r, and ξ_a are the probabilities of noise-induced distortion of a DATA frame including a fragment of length ℓ_f ($\xi_d(\ell)$), RTS frame (ξ_r), and CTS and ACK frames (ξ_a) of identical format [1]. (That is, ξ_{rc} and $\xi(l_f)$ are the probabilities of distorting an RTS-CTS and DATA-ACK sequences, respectively.) These distortion probabilities are defined by the Bit Error Rate (BER)—the probability of distortion of a bit, i.e., an f-bit frame is distorted with probability $\xi_f = 1 - \exp\{-f\mathrm{BER}\}$. (We consider only the case when a BER is constant at least in the course of one packet transmission process.)

The mean duration of attempt to transfer the packet remainder described by the pair (ℓ, k) is obviously equal to

$$
\begin{aligned}
t_s(\ell,k) = Q(r^1_{\ell k}) \sum_{i=0}^{k} (t_{d,i} + \mathrm{SIFS} + \delta)[1 - \xi(L_f)]^i \\
+ Q(r^1_{\ell k})(t_{ACK} + \mathrm{SIFS} + \delta) \sum_{i=0}^{k} (1 - \xi_{d,i})[1 - \xi(L_f)]^i \\
+ \mathbf{1}(r^1_{\ell k} > \overline{P})[t_{RTS} + \delta + \mathrm{SIFS} + (1 - \xi_r)(t_{CTS} + \delta + \mathrm{SIFS})] \\
- \mathrm{SIFS} + \pi_h(\ell,k)\mathrm{DIFS} + [1 - \pi_h(\ell,k)]\mathrm{EIFS},
\end{aligned}
\tag{6}
$$

where

$$
t_{d,i} = \begin{cases} t_d(r^0_\ell), & i = k \\ t_d(L_f), & i < k, \end{cases} \qquad \xi_{d,i} = \begin{cases} \xi_d(r^0_\ell), & i = k \\ \xi_d(L_f), & i < k, \end{cases}
$$

$t_{CTS} = t_{ACK}$ is the transfer time of a CTS and an ACK frame.

Thus, the mean duration T_s of a "successful" slot is

$$
T_s = \sum_{\ell=1}^{\ell_{\max}} \sum_{k=0}^{K(\ell)} t_s(\ell,k) \widehat{d}_{\ell k}. \tag{7}
$$

Therefore, the throughput S can be found by (5)–(7) if the transmission commencement probability τ and the probability distribution $\{\widehat{d}_{\ell k}\}$ are known.

4 Transmission and Rejection Probabilities

Everywhere in this section, we study the process of transmitting a packet of length ℓ by some station. This process starts at the instance when the packet is chosen from the queue and ends with either this packet successful transmission or its rejection. Let f_ℓ and $\overline{w}_\ell$ be the mean numbers of this packet transmission attempts and virtual slots in which the considered station defers from transmission during this process. (We call $\overline{w}_\ell$ the mean backoff time sum.) Then

$$\tau = \sum_{\ell=1}^{\ell_{\max}} d_\ell f_\ell / \sum_{\ell=1}^{\ell_{\max}} d_\ell (f_\ell + \overline{w}_\ell). \tag{8}$$

Moreover, we will seek also the averaged probability $\overline{p}_{rej}$ of packet rejection taking place when one of the counters n_s or n_d attains its limiting value N_s or N_d, respectively. The probability is found from the following sum:

$$\overline{p}_{rej} = \sum_{\ell=1}^{\ell_{\max}} d_\ell p_{rej}(\ell), \tag{9}$$

where $p_{rej}(\ell)$ is the probability of rejecting a packet of length ℓ.

4.1 Short Single-Fragment Packet Transmission

First we consider a simple case $\ell \leq \max\{\overline{P}, L_f\}$ when a packet is not divided into fragments and is transferred by the Basic Access Mechanism. (Note that the difference of the standard and modified backoff rules is not displayed here.) In this case, the number i of packet transmission attempts is bounded by N_s, so we have

$$f_\ell = \sum_{i=1}^{N_s} i\psi_\ell(i), \quad \overline{w}_\ell = \sum_{i=1}^{N_s} \overline{W}_i \psi_\ell(i), \tag{10}$$

where $\psi_\ell(i)$ is the probability that exactly i attempts take place. The mean backoff time sum $\overline{W}_i$ taken under condition that exactly i attempts take place is obtained by (1): with $1 \leq i \leq m+1$

$$\overline{W}_i = \sum_{k=0}^{i-1} \frac{W_k - 1}{2} = W_{i-1} - \frac{W_0 + i}{2}, \tag{11}$$

while with $i > m+1$

$$\begin{aligned} \overline{W}_i &= \sum_{k=0}^{m} \frac{W_k - 1}{2} + \frac{W_m - 1}{2}(i - 1 - m) \\ &= W_m \frac{i - m + 1}{2} - \frac{W_0 + i}{2}. \end{aligned} \tag{12}$$

Let us look for probabilities $\psi_\ell(i),\ i = 1,\ldots,N_s$. The probability of unsuccessful attempt is

$$\pi_{cd}(\ell) = 1 - (1-\pi_c)[1-\xi(\ell)],$$

where $\pi_c = 1-(1-\tau)^{N-1}$ is the probability of the current attempt collision. Then the process is completed successfully at the ith attempt with probability

$$\psi^s_\ell(i) = [1-\pi_{cd}(\ell)][\pi_{cd}(\ell)]^{i-1}, \quad i = 1,\ldots,N_s, \tag{13}$$

or ends in rejection after the N_sth attempt with probability

$$p_{rej}(\ell) = [\pi_{cd}(\ell)]^{N_s}. \tag{14}$$

Hence,

$$\psi_\ell(i) = \psi^s_\ell(i) = [1-\pi_{cd}(\ell)][\pi_{cd}(\ell)]^{i-1}, \quad i = 1,\ldots,N_s-1, \tag{15}$$

$$\psi_\ell(N_s) = \psi^s_\ell(N_s) + p_{rej}(\ell) = [\pi_{cd}(\ell)]^{N_s-1}. \tag{16}$$

4.2 Long Single-Fragment Packet Transmission

Now let $L_f \geq \ell > \overline{P}$. In this case, the number of DATA frame transfers is bounded by N_d and each of these transfers may be preceded by $0,\ldots,N_s-1$ unsuccessful attempts of transferring an RTS frame. Moreover, in the case of a packet rejection due to attaining the limit N_s, the packet transmission process completes with N_s failed RTS transfers.

Let exactly i_d and i_r retries of transferring DATA and RTS frames, respectively, take place in the course of transmitting a packet of length ℓ. Since the probabilities of unsuccessful transfer of DATA and RTS frames are $\xi(\ell)$ and $\pi_{cr} = 1-(1-\pi_c)(1-\xi_{rc})$, respectively, then the transmission process is completed successfully (case $\mathcal{S}_{i_d,i_r}$) with probability

$$\zeta^s_\ell(i_d,i_r) = A_\ell(i_d,i_r)g(i_r,i_d+1), \tag{17}$$

$$i_d < N_d, \quad i_r \leq (N_s-1)(i_d+1),$$

where

$$A_\ell(i_d,i_r) = (1-\pi_{cr})^{i_d+1}[1-\xi(\ell)][\xi(\ell)]^{i_d}\pi_{cr}^{i_r},$$

or ends in packet rejection due to attaining either the limit N_s (case $\mathcal{R}_{i_d,i_r}$) with probability

$$\zeta^r_\ell(i_d,i_r) = \frac{A_\ell(i_d,i_r)g(i_r-N_s,i_d)}{(1-\pi_{cr})[1-\xi(\ell)]}, \tag{18}$$

$$i_d < N_d, \quad i_r - N_s \le (N_s - 1)i_d,$$

or the limit N_d (case $\mathcal{D}_{i_r}$) with probability

$$\zeta_\ell^d(i_r) = A_\ell(N_d - 1, i_r)\frac{\xi(\ell)}{1-\xi(\ell)}g(i_r, N_d), \quad i_r \le (N_s - 1)N_d. \tag{19}$$

In (17)-(19), integer function $g(u, v)$ is the number of ways in which u indistinguishable balls (failed RTS transfers) can be placed in v urns (gaps preceding each of DATA transfers) so that every urn contains not more than $N_s - 1$ balls. This function is computed recursively:

$$g(0, v) = 1 \quad \forall v \ge 0, \quad g(u, 1) = \begin{cases} 1, & u < N_s \\ 0, & u \ge N_s, \end{cases}$$

$$g(u, v) = \sum_{k=0}^{\min(u, N_s-1)} g(u - k, v - 1), \;\; v \ge 2, \; u > 0.$$

Therefore, the probability that exactly i attempts take place is

$$\psi_\ell(i) = \psi_\ell^s(i) + \mathbf{1}(i \ge N_s) \sum_{i_d=0}^{\min\{i-N_s, N_d-1\}} \zeta_\ell^r(i_d, i - i_d)$$

$$+\mathbf{1}(i \ge N_d)\zeta_\ell^d(i - N_d), \quad i = 1, \ldots, i_m^1, \tag{20}$$

where

$$\psi_\ell^s(i) = \sum_{i_d=0}^{\min\{i, N_d\}-1} \zeta_\ell^s(i_d, i - i_d - 1) \tag{21}$$

is the probability that the process is completed successfully at the ith attempt, and $i_m^1 = (N_s - 1)N_d + 1$ is the maximal number of attempts carried out for transmitting a long single-fragment packet.

Now we obtain the mean number of attempts (f_ℓ) by the first equation of (10), where i_m^1 is substituted for N_s.

Moreover, the rejection probability for a packet of length ℓ is

$$p_{rej}(\ell) = \sum_{i_d=0}^{N_d-1} \sum_{i_r=N_s}^{N_s+i_d(N_s-1)} \zeta_\ell^r(i_d, i_r) + \sum_{i_r=0}^{N_d(N_s-1)} \zeta_\ell^d(i_r). \tag{22}$$

To complete this case consideration, it remains to find the mean backoff time sums $\overline{w}_\ell$ for all ℓ, and this is the point where the difference of backoff rules is displayed. Since n_r-counter increases by 1 with each failure, so $\overline{w}_\ell$ is determined by the second equation of (10), where i_m^1 is substituted for N_s.

4.3 Multiple-Fragment Packet Transmission

In the course of transmitting a fragmented packet of length $\ell > L_f$, attempts of transferring the packet remainder described by the pair (ℓ, k), $k = 0, \dots, K(\ell) - 1$, may take place if and only if

(i) all preceding fragments $1, \dots, K(\ell) - k$ had been successfully transferred, i.e., the n_s- and n_ℓ-counters had not attained their limiting values, and

(ii) in the first attempt of transferring the fragment $K(\ell) - k + 1$ being the start of this remainder, either the DATA frame containing the fragment or the corresponding ACK frame was distorted, which resulted in advancing either n_s- or n_d-counter (depending on this fragment length) by one.

The probability $z_{\ell k}$ that both of these conditions are satisfied is determined by the following equations: $z_{\ell,K(\ell)} = 1$ and for $k < K(l)$

$$z_{\ell,k} = \xi(r^1_{\ell k})[1 - p_{rej}(L_f)][1 - \xi(L_f)\widehat{p}_{rej}(L_f)]^{K(\ell)-k-1},$$

where $\widehat{p}_{rej}(r), \quad r = 1, \dots, L_f$, is the rejection probability found either by (14) with substituting $N_s - 1$ for N_s if $r \le \overline{P}$ or by (22) with substituting $N_d - 1$ for N_d in (19) and (22) if $r > \overline{P}$. (We take into account of the condition (ii) by this substitution.)

Moreover, for $\ell > L_f$ the rejection probability is

$$p_{rej}(\ell) = p_{rej}(L_f) + \sum_{k=1}^{K(\ell)-1} \widehat{p}_{rej}(L_f) z_{\ell,k} + \widehat{p}_{rej}(r^0_\ell) z_{\ell,0}. \tag{23}$$

Now let us find the probability $\psi_\ell(i)$ (for $\ell > L_f$) that exactly i attempts take place during the considered process of transmitting a packet of length ℓ. We have

$$\begin{aligned} \psi_\ell(i) = &\mathbf{1}[i \le i_m(L_f)][\psi_{L_f}(i) - \psi^s_{L_f}(i)] \\ &+ \sum_{h=1}^{\min[i, i_m(L_f)]} \psi^s_{L_f}(h) G_{K(\ell)-1}(\ell, i - h), \end{aligned} \tag{24}$$

where the first item of (24) is the probability of the packet rejection with transmitting the first fragment,

$$i_m(r) = \begin{cases} N_s & \text{if } r \le \overline{P} \\ (N_s - 1)N_d + 1 & \text{if } r > \overline{P}, \end{cases}$$

while $G_k(\ell, j)$ is the probability that exactly j attempts take place during transmitting the last $k+1$ fragments under the condition that the first $K(\ell)-k$ fragments have been transmitted successfully.

Obviously, $G_k(\ell, j)$ can be calculated recursively: for $k > 0$

$$G_k(\ell,i) = [1-\xi(L_f)]G_{k-1}(\ell,i) + \mathbf{1}(0 < i \le i^*_m(L_f))\xi(L_f)[\widehat{\psi}_{L_f}(i) - \widehat{\psi}^s_{L_f}(i)]$$

$$+\mathbf{1}(i>0)\xi(L_f)\sum_{h=1}^{\min[i,i_{m1}(L_f)]}\widehat{\psi}^s_{L_f}(h)G_{k-1}(\ell,i-h) \qquad (25)$$

and

$$G_0(\ell,i) = \mathbf{1}(i=0)[1-\xi(r^0_\ell)] + \mathbf{1}(0 < i \le i^*_m(r^0_\ell))\xi(r^0_\ell)\widehat{\psi}_{r^0_\ell}(i),$$

where

$$i^*_m(r) = \begin{cases} N_s - 1 & \text{if } r \le \overline{P} \\ (N_s-1)(N_d-1)+1 & \text{if } r > \overline{P}, \end{cases}$$

while the probabilities $\widehat{\psi}_r(i)$ and $\widehat{\psi}^s_r(i)$ are found similarly to $\psi_r(i)$ and $\psi^s_r(i)$, that is, either by (13), (15), and (16) with substituting N_s-1 for N_s if $r \le \overline{P}$ or by (20) and (21) with substituting N_d-1 for N_d in (19)–(21) if $r > \overline{P}$. In (25), the first item is the conditional probability that no attempts happen with transmitting the fragment $K(\ell)-k+1$ (that is, this fragment is not distorted by noise), so that all i attempts take place during the transmission of the last k fragments. The second item is the conditional probability that all i attempts happen with transmitting the fragment $K(\ell)-k+1$, which results in the packet rejection. The last item corresponds to the case when the fragment $K(\ell)-k+1$ is transferred successfully after $h > 0$ attempts.

Now we can find the average number f_ℓ of attempts for both considered backoff rules as well as the mean backoff time sum $\overline{w}_\ell$ by (10), where N_s is replaced by

$$I_m(\ell) = i_m\left(r^1_{\ell,K(\ell)}\right) + [(K(\ell)-1]i^*_m(L_f) + i^*_m(r^0_\ell),$$

that is the maximal number of attempts for a packet of length ℓ.

Thus, we have derived algebraically complete set of equations to find the value of transmission probability τ.

4.4 Distribution of Attempts and Iterative Procedure

To use the formulas (5), (4) and (7) determining the values of S, T_c and T_s, it remains to find the distribution $\{\widehat{d}_{\ell k}\}$. Obviously,

$$\widehat{d}_{\ell k} = d_\ell f_{\ell k} / \sum_{u=1}^{\ell_{\max}} \sum_{v=0}^{K(u)} d_u f_{uv}, \quad \ell = 1, \ldots, \ell_{\max}, \quad k = 0, \ldots, K(\ell), \tag{26}$$

where $f_{\ell k}$ is the mean number of attempts of transferring the packet remainder described by pair (ℓ, k). For a non-fragmented packet when $\ell \leq L_f$, we have $f_{\ell 0} = f_\ell$ where f_ℓ is determined by the first equation of (10) (with substituting i_m^1 for N_s if $\ell > \overline{P}$), while for $\ell > L_f$ this mean number of attempts is determined as follows:

$$f_{\ell, K(\ell)} = \sum_{i=1}^{i_m(L_f)} i\psi_{L_f}(i), \quad f_{\ell k} = z_{\ell k} \sum_{i=1}^{i_m^*(r_{\ell k}^1)} i\widehat{\psi}_{r_{\ell k}^1}(i), \quad k < K(\ell). \tag{27}$$

We adopt the following iterative procedure to estimate the transmission probability τ.

Step 0. Define an initial value for τ.

Step 1. For packet lengths $\ell = 1, \ldots, L_f$, compute the probabilities $\psi_\ell(i)$ and $\psi_\ell^s(i)$ for $i = 1, \ldots, i_m(\ell)$, the rejection probability $p_{rej}(\ell)$, the mean number of attempts (f_ℓ), and the mean backoff time sum $\overline{w}_\ell$, using (10), and (13)–(16) if $\ell \leq \overline{P}$. If $\ell > \overline{P}$ use (17)–(22) and the modified equations (10) with substituting i_m^1 for N_s.

Step 2. For all $\ell = 1, \ldots, L_f$, compute the modified probabilities $\widehat{\psi}_\ell(i)$, $\widehat{\psi}_\ell^s(i)$ $(i = 1, \ldots, i_m^*(\ell))$ and $\widehat{p}_{rej}(\ell)$ by the same formulas as for $\psi_\ell(i)$, $\psi_\ell^s(i)$ and $p_{rej}(\ell)$, in which N_s is replaced by $N_s - 1$ if $\ell \leq \overline{P}$ or $N_d - 1$ is substituted for N_d if $\ell > \overline{P}$.

Step 3. For all $L_f < \ell \leq l_{max}$, compute the probability $\psi_\ell(i)$, $i = 1, \ldots, I_m(\ell)$, by (24), the mean number of attempts (f_ℓ) and the mean backoff time sum $\overline{w}_\ell$ by the modified equations (10) with substituting $I_m(\ell)$ for N_s.

Step 4. Using (8), find the modified value of τ and compare it with the initial value. If the difference of these values is greater than a predefined limit, return to Step 1, using a new initial value for τ—the half-sum of its old initial value and the modified value.

After this iterative procedure, we obtain the averaged rejection probability $\overline{p}_{rej}$ by (10), (14), (22), and (23). Finally, we find the distribution $\{\widehat{d}_{\ell k}\}$ by (26)–(27) and the throughput S by the formulas of Section 3.

We don't prove exactly the convergence of this iterative technique due to its complexity. It is clear intuitively that the equation (8) has a unique solution because a growth of transmission probability τ leads to increasing the collision probability and, hence, to increasing the average number $\overline{w}_\ell/f_\ell$ of slots anticipating an attempt for all ℓ. In practice, numerous examples of adopting our technique with various values of Wi-Fi LAN parameters have shown that this technique provides very fast convergence to the solution and high speed of calculating the values of estimated performance indices.

5 Conclusions

In this paper, we have developed an analytical method for estimating the throughput of a Wi-Fi LAN operating under saturation and in the presence of noise. Besides the throughput, our method allows evaluating the probability of a packet rejection due to the attainment of the limiting values specified by the Standard [1] for the number of retries for transferring long and short frames. Unlike previous works [7, 8, 6], and [7], this method is useful in estimating the 802.11 LAN performance indices under packet fragmentation recommended in Standard [1] for reducing the influence of noise. Comparing numerical results obtained by both the developed method and GPSS simulation [5], we have shown that our method is quite exact: the errors never exceed 2% with throughput estimation and 5% with rejection probability estimation. Moreover, this method provides a high speed of calculating the values of performance indices, which has allowed us to adopt the exhaustive search of optimal RTS and fragmentation thresholds and to develop some recommendations on tuning these protocol parameters.

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