Hybrid RTS/CTS mechanism in Wi-Fi Ad Hoc Networks with Correlated Channel Failures

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Abstract— **In this paper, we present an analytical method for estimating the saturation throughput of an 802.11 ad hoc LAN called also the Wi-Fi (Wireless Fidelity) LAN in the presence of correlated channel failures usually inherent to wireless channels. With the study, we consider the hybrid RTS/CTS mechanism, in which a packet is transmitted under the RTS/CTS mechanism if its length is higher than a fixed threshold, and under the Basic Access mechanism otherwise. In addition to the throughput in saturation, our method allows estimating the probability of a packet rejection occurring when the number of packet transmission retries attains its limit. The obtained numerical results of investigating Wi-Fi LANs by the developed method are validated by simulation and show high estimation accuracy as well as the method efficiency in determining the optimal RTS threshold.**

I. Introduction

IEEE 802.11 [12] is one of the most popular technologies for wireless ad hoc and mobile networking. The fundamental access mechanism in the IEEE 802.11 protocol is the Distributed Coordination Function (DCF). The DCF was studied in depth in [1], [3], [8], where analytical methods were developed for evaluating the performance of 802.11 wireless LANs in the saturation conditions, when there are always queues for transmitting at every wireless LAN station. This performance index called the *saturation throughput* in [1] was evaluated in the assumption of ideal channel conditions, i.e., in the absence of noise, causing the throughput overestimation.

There may be different noise sources: other devices located in the LAN neighborhood and operating on the same license-free frequency band, multipath fading, co-/adjacent channel interference, etc. (Detail arguing of noise sources can be found in [11], for example.) In [6] and [9], we have developed the methods of [1], [3], [8] to study the influence of noise on the Wi-Fi LAN performance, assuming channel failures (that is, noise-induced distortions) uncorrelated, for instance, in case of a channel adding white gaussian noise. However, it is known (e.g., see [10] and [13]) that the wireless-medium behavior is better characterized by the Gilbert model [4] representing a two-states Markov chain. There are Good and Bad states, which differ in the Bit Error Rate (BER) being constant in each state. Obviously, according to the model, channel failures caused by noise influence are correlated, and this correlation makes hard the Wi-Fi network performance analysis, forcing previous investigators of the problem to adopt simulation (see [2], for instance). Nevertheless, in this paper, we succeed in studying analytically the performance of the Wi-Fi network with correlated channel failures, assuming that stochastic sojourn times in Bad and Good states are distributed exponentially.

Further in Section II we briefly review the DCF operation in saturation and noise. In Sections III–V, we study a packet transmission process in a Wi-Fi ad hoc LAN with correlated channel failures under a hybrid RTS/CTS mechanism, using the analytical method (proposed firstly in [7]) of estimating the saturation throughput and the probability of a packet rejection occurring when the number of packet transmission retries attains its limit. In Section VI, we give some numerical research results of 802.11 LAN performance evaluation. These results obtained by both our analytical method and simulation allow us to validate the developed method and to show how the correlation of channel failures affects the LAN performance and the RTS/CTS mechanism efficiency. Finally, the obtained results are summarized in Section VII.

II. DCF in Saturation

Now we briefly outline the DCF scheme, considering only the aspects that are exhibited in saturation and with absence of hidden stations. This scheme is described in detail in [12].

Under the DCF, data packets are transferred in general via two methods (for this reason, this mechanism is referred as hybrid). Short packets of length not greater than the limit $\overline{P}$ (called the RTS threshold in [12]) are transferred by the Basic Access Mechanism (BAM). In this mechanism shown in Figure 1, a station confirms the successful reception of a DATA frame by a positive acknowledgment ACK after a short SIFS interval (which will be noted δ_S in what follows).

Packets of length greater than $\overline{P}$ are transferred via the Request-To-Send/Clear-To-Send (RTS/CTS) mechanism. In this case shown in Figure 2, first an

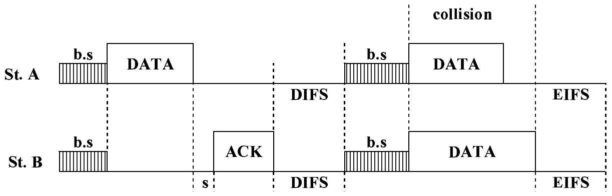

Figure 1. BAM (s - SIFS, b.s - backoff slots)

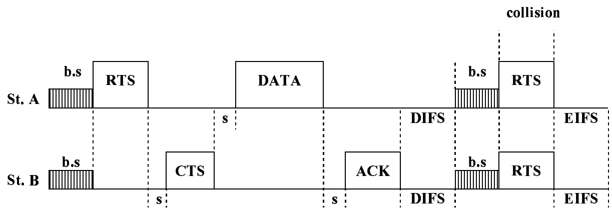

Figure 2. RTS/CTS mechanism

inquiring RTS frame is sent to the receiver station, which replies by a CTS frame after a SIFS. Then only a DATA frame is transmitted and its successful reception is confirmed by an ACK frame. Since there are no hidden stations in the considered LAN, all other stations hear the RTS frame transmission and defer from their own attempts. This protects CTS, DATA and ACK frames from a collision-induced distortion. Thus the RTS threshold $\overline{P}$ is chosen as a result of a reasonable trade-off between the RTS/CTS mechanism overhead consisting in transmitting two additional control frames (RTS and CTS) and reduction of collision duration. Figures 1 and 2 show that the collision duration is determined by the length of the longest packet involved in collision for the BAM, whereas in the RTS/CTS mechanism it is equal to the time of transferring a short RTS frame.

After a packet transfer attempt the station passes to the backoff state after a DIFS interval (δ_D in what follows) if the attempt was successful (i.e., there was no collision, all frames of a packet were transferred without noise-induced distortions) or after an EIFS (δ_E in what follows) interval if the attempt failed. The backoff counter is reset to the initial value b, which is called the backoff time, measured in units of backoff slots of duration σ, and chosen uniformly from a set $(0, \ldots, w-1)$. The value w, called the contention window, depends on the number n_r of attempts performed for transmitting the current packet: $w = W_{n_r}$, where

$$W_{n_r} = \begin{cases} W_0 2^{n_r} \text{ for } n_r \leq N_r \\ W_{n_r} = W_0 2^{N}_{r} \text{ for } n_r > N_r \end{cases} . \quad (1)$$

Backoff interval is reckoned only as long as the channel is free: the backoff counter is decreased by one only if the channel was free in the whole previous slot. Counting the backoff slots stops when the channel becomes busy, and backoff time counters of all stations can decrement next time only when the channel is sensed idle for the duration of σ+DIFS or σ+EIFS if the last sensed transmission is successful or failed, respectively. When the backoff counter attains its zero value, the station starts transmission.

In the course of transmission of a packet, a source station counts the numbers of short (n_s) and long (n_ℓ) retries. Let a source station transfer a DATA frame with a packet of length equal to or less than $\overline{P}$, or an RTS frame. If a correct ACK or CTS frame, respectively, is received within timeout, then the n_s-counter is zeroed; otherwise n_s is advanced by one. Similarly, the n_ℓ-counter is zeroed or advanced by one in case of reception or absence of a correct ACK frame (within timeout) confirming the successful transfer of a DATA frame with a packet of length greater than $\overline{P}$. When any of these counters n_s and n_ℓ attains its limit N_s or N_ℓ respectively, the current packet is rejected. After the rejection or success of a packet transmission the next packet is chosen (due to saturation) with zeroing the values of n_r, n_s, and n_ℓ.

III. Analytical Study

A. Model

Let us consider a small-size Wi-Fi ad hoc LAN of N statistically homogeneous stations working in saturation. In fact, we mean by N not a number of all stations of the LAN, but a number of active stations, whose queues are not empty for a quite long observation interval. By statistical homogeneity of stations, we mean the following: (i) the lengths of packets (in bytes) chosen by every station from the queue have an identical probability distribution $\{d_\ell,\ \ell = \ell_{min}, \ldots, \ell_{max}\}$; (ii) all stations adopt the same RTS threshold $\overline{P}$; (iii) since the distance between stations is small, we neglect the propagation delay and assume that there are no hidden stations and noise occurs concurrently at all stations. The last assumption implies that all stations "sense" the common wireless channel identically.

Also, to describe the channel state change, we adopt the two-states Gilbert model [4] modified as follows: the channel stays in state $i \quad (i = 0, 1)$ during a time interval distributed exponentially with parameter λ_i. The channel states differ in BER. More precisely, in state i, BER is equal to $\mu_i^h/8$ and $\mu_i/8$ with transmitting a PHY h-byte header and the other frame part, respectively, and an $h+f$-byte frame transmitted entirely with the channel state i is distorted with probability $1 - \exp\{-\mu_i^h h - \mu_i f\}$. We have to adopt different BERs, since PHY headers are usually transmitted with a lower channel rate, but more reliable coding and modulation scheme. The channel state change rates λ_0 and λ_1 are assumed to be not too high, so that no more than one state change can happen during a frame transmission or an interframe space.

As in [1], [7] and [9], let us subdivide the time of the LAN operation into non-uniform virtual slots such that every station changes its backoff counter at the start of a virtual slot and can begin transmission if the value of the counter becomes zero. Such a virtual slot is either (a) an "empty" slot in which no station transmits, or (b) a "successful" slot in which one and only one station transmits, or (c) a "collisional" slot in which two or more stations transmit.

As in [1], [3], [7] and [9], we assume that the probability that a station starts transmitting a packet in a given slot does not depend neither on the previous history, nor on the behavior of other stations, and is equal to τ_i, which is the same for all stations and depends only on the current channel state i. Hence the probabilities $P^c_{i,k}$, $i \in \{0,1\}$, $k \in [0,N]$ that, in an arbitrarily chosen virtual slot starting when the channel is in state i, k stations try to transmit are $P^c_{i,k} = C^k_N \tau^k_i (1-\tau_i)^{N-k}$. Consequently, an arbitrarily chosen virtual slot is "empty" with probability $p^i_e = P^c_{i,0}$ or "successful" with probability $p^i_s = P^c_{i,1}$.

Finally, we note ψ^S_i the probability that a given slot begins when the channel is in state i and we make the following approximation: $\psi^S_i \approx \lambda_{i^*}/(\lambda_0+\lambda_1)$ (here and in what follows, $i^*=1$ with $i=0$ and $i^*=0$ with $i=1$).

B. Throughput Evaluation

The throughput S is defined as the average number of successfully transferred payload bits per second. Obviously,

$$S = \sum_{i=0}^{1} \psi^S_i S_i, \qquad (2)$$

where S_i is the throughput observed when the channel is in state i. S_i is determined by the formula

$$S_i = \frac{p^i_s U^i}{p^i_s T^i_s + p^i_e \sigma + \sum_{k=2}^{N} P^c_{i,k} T^c_{i,k}} \qquad (3)$$

where U^i is the mean number of successfully transferred data bytes in a "successful" slot starting in state i, T^i_s is the mean duration of a "successful" slot beginning in state i, and $T^c_{i,k}$ is the mean duration of a collisionnal slot involving k stations and beginning in state i.

The duration of a "collisional" slot is the sum of time of transmitting the longest frame involved in collision and an EIFS interval:

$$T^c_{i,k} = t_{RTS}\big(\sum_{\ell=\overline{P}+1}^{\ell_{max}} \widehat{d^i_\ell}\ \big)^k + \sum_{\ell=\ell_{min}}^{\overline{P}} t_d(\ell) P_i(k,\ell) + \delta_E \quad (4)$$

where:

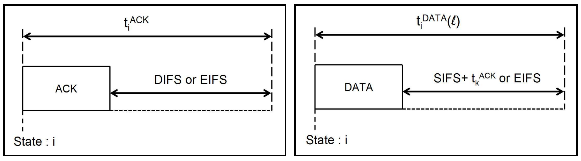

Figure 3. Computational method corresponding to equations (6)–(7)

- t_{RTS} is the transfer time for an RTS frame
- $\widehat{d^i_\ell}$ is the probability that the performed attempt, starting when the channel is in state i, is related to a packet of length ℓ. Note that the distribution $\{\widehat{d^i_\ell}\}_{\ell,i}$ is different from the distribution $\{d_\ell\}_\ell$ (the longer the length of a packet, the greater $\widehat{d^i_\ell}$).
- $t_d(\ell) = H + H_{MAC} + \ell/V$ is the transmission time of a DATA frame including a packet of length ℓ, the PHY header is transmitted in time H and the MAC header is transmitted in time H_{MAC}, V is the channel rate (in bytes per a second) (according to [12], $t_{RTS} < H + H_{MAC}$, so that we always have $t_d(\ell) > t_{RTS}$)
- $P_i(k,\ell)$ is the probability that the duration of the considered collision is $t_d(\ell)$. Defining $S_i(\ell_0) = \sum_{y=\ell_{min}}^{\ell_0-1} \widehat{d^i_y} + \sum_{y=\overline{P}+1}^{\ell_{max}} \widehat{d^i_y}$ we have $P_i(k,\ell) = \big(S_i(\ell+1)\big)^k - \big(S_i(\ell)\big)^k$

The case of a "successfull" is more complicated, and we study it in the next section.

IV. Successful Slot

The mean duration T^i_s and the mean number U^i are given by

$$T^i_s = \sum_{\ell=\ell_{min}}^{\ell_{max}} t^i_s(\ell)\ \widehat{d^i_\ell}\ , \quad U^i = \sum_{\ell=\ell_{min}}^{\ell_{max}} \ell\ \widehat{d^i_\ell}\ \rho^{SLOT}_i(\ell)$$

where $t^i_s(\ell)$ is the mean duration of a "successfull" slot related to a packet of size ℓ and starting in state i, and $\rho^{SLOT}_i(\ell)$ is the probability that no distortion occurs during a "successful" slot starting in state i and related to a packet of size ℓ.

We can define $t^{DATA}_i(\ell)$ and $t^{RTS}_i(\ell)$ so that we have :

$$t^i_s(\ell) = \begin{cases} t^{DATA}_i(\ell) \text{ if } \ell \le \overline{P} \\ t^{RTS}_i(\ell) \text{ if } \ell > \overline{P} \end{cases} \qquad (5)$$

In order to do that, let us introduce a computational method that we will use several times (see Figure 3).

First, we begin by defining t^{ACK}_i, the mean duration of the end of a "successfull" slot, when the channel is in state i and is about to send an ACK frame:

$$t^{ACK}_i = t_{ACK} + \delta_D \rho_i(\ell_{ACK}) + \delta_E\big(1-\rho_i(\ell_{ACK})\big) \quad (6)$$

where $\rho_i(f)$ is the probability that a MAC frame of length f is not distorted. Then, we define $t^{DATA}_i(\ell)$

as the mean duration of the interval of time elapsed from the moment when the channel, in state i, begins to transfer a DATA frame with a packet of length ℓ, and the end of the "successfull" slot during which this attempt takes place:

$$t_i^{DATA}(\ell) = t_d(\ell) + \delta_S \rho_i(\ell_m) + \delta_E\big(1 - \rho_i(\ell_m)\big) + \sum_{k=0}^{1} \nu_{ik}^0(\ell_m)\Big(\sum_{k'=0}^{1} \gamma_{kk'}(\delta_S) t_{k'}^{ACK}\Big) \quad (7)$$

where $\nu_{ij}^0(f)$ is the probability that a MAC frame of length f is not distorted and that the channel, in state i at the beginning of this transmission, ends in state j. Moreover, $\ell_m = \ell + h_{MAC}$ is the DATA frame length, including the MAC header. $\gamma_{ij}(t)$ is the probability that the channel passes from state i to j during an interval of time of size t: $\gamma_{ii}(t) = 1 - \gamma_{ii^*}(t) = e^{-\lambda_i t}$.

We obtain by the same way a formula for $t_i^{CTS}(\ell)$ by replacing in (7) $t_d(\ell)$ by t_{CTS}, ℓ_m by ℓ_{CTS} and t_k^{ACK} by $t_k^{DATA}(\ell)$. And with this formula for $t_i^{CTS}(\ell)$, we define and compute by the same idea $t_i^{RTS}(\ell)$.

With these definitions and formulas, we do effectively have the relations announced in (5). We still have to find out $\rho_i(f)$ and $\nu_{ij}^0(f)$. Obviously, $\rho_i(f) = \nu_{ii}^0(f) + \nu_{ii^*}^0(f)$, and we have

$$\nu_{ii}^0(f) = \exp\{-\lambda_i(H + f/V) - \mu_i^h h - \mu_i f\}$$

$$\nu_{ii^*}^0(f) = e^{-\mu_{i^*} f} I_i^h + e^{-\mu_i^h h - \lambda_i H} I_i(f),$$

where

$$\begin{aligned} I_i^h &= \int_0^H \lambda_i e^{-\lambda_i t} \exp\{-V_h[\mu_i^h t + \mu_{i^*}^h (H - t)]\}dt \\ &= e^{-\mu_{i^*}^h h} \tfrac{\lambda_i H}{\lambda_i H + h(\mu_i^h - \mu_{i^*}^h)} [1 - e^{-\lambda_i H - h(\mu_i^h - \mu_{i^*}^h)}] \end{aligned}$$

with $\lambda_i H \neq h(\mu_{i^*}^h - \mu_i^h)$ and $I_i^h = \lambda_i H e^{-\mu_{i^*}^h h}$ otherwise. $I_i(f)$ is defined similarly with the substitution of f for h, f/V for H, and μ_i and μ_{i^*} for μ_i^h and $\mu_{i^*}^h$, respectively. In what follows, we will also need the $\nu_{ij}^1(f)$, which are defined as the $\nu_{ij}^0(f)$ but in case of a failure of the transmission due to distortion:

$$\nu_{ii}^1(f) = \exp\{-\lambda_i(H + f/V)\}[1 - \exp\{-\mu_i^h h - \mu_i f\}]$$

$$\nu_{ii^*}^1(f) = 1 - \nu_{ii}^0(f) - \nu_{ii^*}^0(f) - \nu_{ii}^1(f).$$

To compute U^i, we need to compute $\rho_i^{SLOT}(\ell)$. That for, we define $\rho_i^{DATA}(\ell)$ and $\rho_i^{RTS}(\ell)$ as in (6)-(7) so that we have :

$$\rho_i^{SLOT}(\ell) = \begin{cases} \rho_i^{DATA}(\ell) \text{ if } \ell \leq \overline{P} \\ \rho_i^{RTS}(\ell) \text{ else} \end{cases}$$

To do so, we only consider the right double sum of the formula (7), replacing in it t_k^{ACK} by $\rho_i(\ell_{ACK})$ and $t_k^A(\ell)$ by $\rho_k^A(\ell)$ (A being either DATA or CTS).

Thus, we have found all components of (3), if the transmission beginning probabilities τ_0 and τ_1 and the probability distribution $\{\widehat{d_\ell^i}\}$ are known.

V. Transmission and Rejection Probabilities

To determine τ_i and $\widehat{d_\ell^i}$, we consider one particular station, and we look at it during the whole process of transmission of a packet (a process begins with the selection of a packet and ends with either the rejection of this packet, or with its successfull transfer). Then, noting ℓ the length of the packet that the station has chosen, we define f_ℓ^i as the mean number of the packet transmission attempts during this process and ω_ℓ^i as the mean number of virtual slots in which the condidered station defers from transmission during this process (each time, these attempts and slots are taken into account only if the channel is in state i at their beginnings). Then:

$$\tau_i = \frac{\sum_{\ell=\ell_{min}}^{\ell_{max}} d_\ell f_\ell^i}{\sum_{\ell=\ell_{min}}^{\ell_{max}} d_\ell (f_\ell^i + \omega_\ell^i)} \quad (8)$$

$$\widehat{d_\ell^i} = \frac{d_\ell f_\ell^i}{\sum_{k=\ell_{min}}^{\ell_{max}} d_k f_k^i} \quad (9)$$

Let us start with looking for f_ℓ^i. First of all, we define an elementary process as the set of all the consecutive slots between the first slot of a backoff state and the end of the slot during which the considered station has tried to transmit (see figures 1 and 2 where each time two elementary processes are depicted).

Then, in the case $\ell > \overline{P}$, we define the function $F_{i,\ell}^L(i_s, n_s, n_l, n_r)$ as the mean number of the packet transmission attempts during a set of elementary processes such that:

- we consider consecutive elementary processes until either the successfull transfer of the packet, or its rejection
- at the beginning of the first slot of the first elementary process, the parameters of the station are equal to n_s, n_l and n_r, and the channel is in state i_s

With such a definition (and by doing the same in the case $\ell \leq \overline{P}$), we can write f_ℓ^i in the following form:

$$f_\ell^i = \begin{cases} \sum_{i_s=0}^{1} \psi_{i_s}^p F_{i,\ell}^S(i_s, 0, 0) \text{ if } \ell \leq \overline{P} \\ \sum_{i_s=0}^{1} \psi_{i_s}^p F_{i,\ell}^L(i_s, 0, 0, 0) \text{ else} \end{cases} \quad (10)$$

where ψ_i^p is the probability that, at the beginning of the first slot a process, the channel is in state i. It is easy to show that

$$\psi_i^p = (1 - \Phi_{i^*})/(2 - \Phi_0 - \Phi_1), \quad (11)$$

where Φ_i is the probability that, at the end of a packet transmission process, the channel appears in the same state i as at the process beginning.

Functions $F^S_{i,\ell}(\cdot)$ and $F^L_{i,\ell}(\cdot)$ are calculated recursively, the iteration concerning an elementary process and its successor.

A. Determination of $F^S_{i,\ell}$

We use the following formula:

$$F^S_{i,\ell}(i_s, n_r, n_s) = \sum_{j=0}^{1} q_j(n_r, i_s)\Bigg\{ 1_{\{j=i\}} + 1_{\{n_s<N_s-1\}}\times \sum_{k=0}^{1}\Big[\alpha^c_{jk}(\ell) + (1-p^{cc}_j)p^{DATA}_{jk}(\ell)\Big] F^S_{i,\ell}(k, n^*_r, n_s+1)\Bigg\} \quad (12)$$

where

- $q_j(n_r, i_s)$ is the probability that the backoff state, beginning in state i_s with parameter n_r, ends in state j. We define $p^{(b)}_{ij}$ as the probability that the backoff state, beginning in state i, ends in state j in b virtual slots. Then

$$q_j(n_r, i_s) = \frac{1}{W(n_r)} \sum_{b=0}^{W(n_r)-1} p^{(b)}_{i_s j}$$

where $\|p^{(b)}_{ij}\| = \|p_{ij}\|^b,\ i, j = 0, 1$, and p_{ij} is the probability that the channel passes from state i to j for a virtual slot, during which the given station does not transmit. The probabilities $\{p_{ij}\}_{(i,j)}$ are computed thanks to a decomposition similar to the one used in the denominator of formula (3), a formula similar to equation (4) and method (6)–(7).

- p^{cc}_i is the probability of the current attempt failure due to a collision: $p^{cc}_i = 1 - (1-\tau_i)^{N-1}$
- $\alpha^c_{ij}(\ell)$ is the probability that the channel, trying to transmit a packet of length ℓ, and collision occuring, passes from state i to state j. Under the condition $\ell \le \overline{P}$, we have $\alpha^c_{ij}(\ell) = \sum_{k=1}^{N-1} \alpha^c_{ij}(k,\ell)\widehat{P}^c_{i,k}$, where $\widehat{P}^c_{i,k}$ is defined as $P^c_{i,k}$ by replacing N by $N-1$, with:

$$\alpha^c_{ij}(k,\ell) = \Big(\textstyle\sum_{k'=0}^{1} \gamma_{ik'}(t_d(\ell))\gamma_{k'j}(\delta_E)\Big)\Big(S_i(\ell+1)\Big)^k + \textstyle\sum_{y=\ell+1}^{\overline{P}}\Big(\sum_{k'=0}^{1}\gamma_{ik'}(t_d(y))\gamma_{k'j}(\delta_E)\Big)P_i(k,y)$$

- $p^{DATA}_{ij}(\ell)$ is the probability that the channel, trying to transmit a DATA frame of length ℓ, and no collision occuring, but distortion occuring, passes from state i to state j. We use method (6)-(7) to compute the $p^{DATA}_{ij}(\ell)$.
- and $n^*_r = \min\{n_r+1, N_r\}$

B. Determination of $F^L_{i,\ell}$

The formula is similar to the precedent one:

$$F^L_{i,\ell}(i_s, n_r, n_s, n_l) = \sum_{j=0}^{1} q_j(n_r, i_s)\Bigg\{ 1_{\{j=i\}} + \sum_{k=0}^{1}\Bigg(1_{\{n_s<N_s-1\}}\Big[\alpha^c_{jk} + (1-p^{cc}_j)\varepsilon^d_{jk}\Big]\times F^L_{i,\ell}(k, n^*_r, n_s+1, n_l) + 1_{\{n_l<N_l-1\}}(1-p^{cc}_j)\times \Big(\sum_{k'=0}^{1}\zeta_{jk'}p^{DATA}_{k'k}(\ell)\Big)F^L_{i,\ell}(k, n^*_r, 0, n_l+1)\Bigg)\Bigg\} \quad (13)$$

where only a few items are new

- ε^d_{jk} is the probability, knowing that no collision occurs, that distortion occurs in RTS or CTS frames and the channel passes from state j to state k. To compute ε^d_{jk}, we use method (6)–(7).
- we have already defined $\alpha^c_{ij}(\ell)$. When $\ell > \overline{P}$: $\alpha^c_{ij}(\ell) = \sum_{k=1}^{N-1}\alpha^c_{ij,k}\widehat{P}^c_{i,k} = \alpha^c_{ij}$ where $\alpha^c_{ij,k}$ is computed thanks to a formula similar to (4).
- and ζ_{ij} is the probability, knowing that no collision occurs, that no distortion occurs in RTS or CTS frames and the channel passes from state i at the beginning of the virtual slot to state j at the end of the SIFS following the CTS frame.

$$\zeta_{ij} = \sum_{k_1,k_2,k_3=0}^{1} \nu^0_{ik_1}(\ell_{RTS})\gamma_{k_1k_2}(\delta_S)\nu^0_{k_2k_3}(\ell_{CTS})\gamma_{k_3j}(\delta_S)$$

To find w^i_ℓ, Φ_i, and $p_{rej}(\ell)$[1] (the probability of rejection of a packet of length ℓ), we first of all write

$$\Phi_i = \sum_{\ell=\ell_{min}}^{\overline{P}} d_\ell\ \phi^S_{i,\ell}(i,0,0) + \sum_{\ell=\overline{P}+1}^{\ell_{max}} d_\ell\ \phi^L_{i,\ell}(i,0,0,0)$$

Then, we define functions $W^A_{i,\ell}(\cdot)$ and $R^A_\ell(\cdot)$ to compute ω^i_ℓ and $p_{rej}(\ell)$ respectively as in (10). And finally, functions $W^A_{i,\ell}(\cdot)$, $\phi^A_{i,\ell}(\cdot)$ and $R^A_\ell(\cdot)$ are defined by (12) (for $A = S$) and (13) (for $A = L$) modified as follows:

- for $W^A_{i,\ell}(\cdot)$, $1_{\{i=j\}}$ is excluded and the item

$$w_i(i_s, n_r) = \sum_{b=1}^{W(n_r)-1} \frac{W(n_r)-b}{W(n_r)} p^{(b-1)}_{i_s i}$$

starts the right part of (12) and (13)

- for $R^A_\ell(\cdot)$ and $\phi^A_{i,\ell}(\cdot)$, the situation is slightly more complex, but each time we replace $1_{\{i=j\}}$ by an item which is different in the short case and in the long case

Thus, we can calculate the throughput S and the averaged packet rejection probability p_{rej}, using the following iterative procedure. Firstly, we calculate all

[1] we find the probability of rejection with $p_{rej} = \sum_{\ell=\ell_{min}}^{\ell_{max}} d_\ell p_{rej}(\ell)$

the values which do not depend on τ_i and $\widehat{d^i_\ell}$ (i.e. the $\nu^0_{i,k}$, $\nu^1_{i,k}$, $\gamma_{jk}(t)$, $t_d(\ell)$, $\cdots$). Then we define initial values for τ_i and $\widehat{d^i_\ell}$ with all possible i, ℓ and calculate their modified values by (8)–(13). If not both relative differences of initial and modified values of τ_i and $\widehat{d^i_\ell}$ are less than a small pre-defined limit, then we set new initial values of τ_i and $\widehat{d^i_\ell}$ equal to halfsums of their modified and old initial values and repeat the calculation.

VI. Numerical Results

To validate our model, we have compared its results with that obtained by a simulation coded in Java and considering all real features of the 802.11 MAC protocol. This comparison has shown a high level of accuracy of the analytical model: the error never exceeds 5% with throughput estimation, and 2% with rejection probability estimation. The object of our numerical investigations was a saturated Wi-Fi ad hoc LAN consisting of N stations. The values of protocol parameters used to obtain numerical results for the analytical model and simulation were the IEEE 802.11b default values [5] for the Long Preamble mode. Moreover, the payload size ℓ is sampled uniformly from the set $\{1, \ldots, 2000\}$.

In what precede, we have described the noise through two sorts of parameters: the λ_i and the μ_i (and μ^h_i). Considering the MAC layer (what follows can be applied as well for the PHY layer, by replacing μ_i by μ^h_i), as the channel spends a time fraction $\frac{\lambda_{i*}}{\lambda_0+\lambda_1}$ in state i, we can define an apparent bit error rate b by the formula

$$8b = \frac{\lambda_1}{\lambda_0+\lambda_1}\mu_0 + \frac{\lambda_0}{\lambda_0+\lambda_1}\mu_1 = m\mu_1 \qquad (14)$$

where $m = (\Gamma + \Lambda)/(1 + \Lambda)$, $\Gamma = \mu_0/\mu_1$ and $\Lambda = \lambda_0/\lambda_1$. This definition makes sense, since when the noise is uncorrelated, i.e. $\Gamma = 1$, we have $8b = \mu_0 = \mu_1$, i.e. the apparent bit error rate is equal to the bit error rate which is the same in both states.

Equation (14) shows that for a given apparent BER, there are infinite ways of obtening it by playing on Λ, Γ and μ_1. In this section, we fix the parameters Γ and Λ with $\Gamma = 1/7$ and $\Lambda = 1/5$ (what gives $m \approx 0.143$) so that if BER denotes the apparent BER, $\mu_0 = 4$ BER and $\mu_1 = 28$ BER. So here, we are just interested in the influences of the BER, of the number of stations and of the RTS threshold.

A. *Correlation fixed: Optimal RTS threshold*

Figure 4 shows the optimal RTS threshold $\overline{P}_{opt}$ maximizing the throughput for varying BER and N, and emphasises on two sorts of curves:

1. for values of BER $< 1.5 \cdot 10^{-4}$ (the exact threshold is somewhere between 10^{-4} and $1.5 \cdot 10^{-4}$), the optimal threshold is constant equal to ℓ_{max} until some threshold N_1. After N_1, $\overline{P}_{opt}$ decreases monotonically with increase of N. This family of curves is not surprising at all: when there are only few stations, the best mechanism is the BAM.
2. for values of BER $> 1.5 \cdot 10^{-4}$, a new threshold $N_0 \leq N_1$ appears: before N_0, $\overline{P}_{opt}$ increases with N. So here we notice the surprising fact that already arose in [9]: when the BER is very high, and the stations are few, the best mechanism is not the BAM, but the RTS/CTS mechanism with a given threshold $\overline{P}_{opt} < \ell_{max}$.

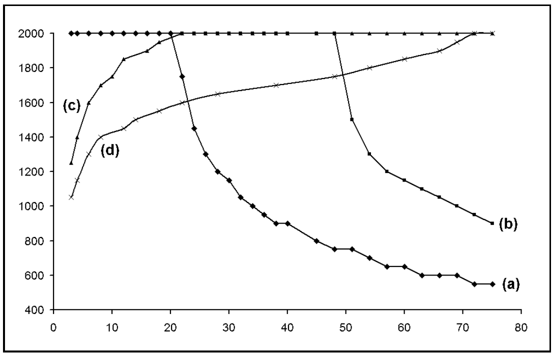

Figure 4. $\overline{P}_{opt}$ vs. N for different values of BER: (a): 10^{-5} (b): 10^{-4} (c): 3.10^{-4} (d): 5.10^{-4}

Let us note that the throughput improvement made by this optimization is significant. For example, when $N = 2$ and BER$= 2 \cdot 10^{-4}$, $S = 1.32$ Mbps under the BAM and $S = 1.41$ Mbps under the RTS/CTS mechanism with $\overline{P} = \overline{P}_{opt} = 1350$. However, the rejection probability is worse under the RTS/CTS mechanism: under the BAM, $p_{rej} = 0.054$, and for $\overline{P} = \overline{P}_{opt}$, $p_{rej} = 0.111$.

We can explain this fact in the same way as in [9]. When there are only few stations and the BER is high, the best mechanism is the RTS mechanism, however the rejection probability is worsened. So in fact, the throughput is improved thanks to a worsening of the rejection probability according to the following explanation. When stations are few, collisions occur only rarely, and a failure is almost always due to distortion. As under the RTS/CTS mechanism, distortion occurs mainly during the DATA or ACK frames, it mainly concerns the long retry counter n_ℓ: in this situation, a station can perform up to $N_s = 7$ retries under the BAM, whereas it mainly perform up to $N_\ell = 4$ retries under the RTS/CTS mechanism. Or, the less maximal the number of attempts (because $N_\ell < N_s$), the larger the rejection probability, the less the mean value of backoff intervals anticipating transmission attempts, and hence the larger the throughput.

B. Correlation analysis

First of all, we must note that the concept of apparent BER is not sufficient. Indeed, for a fixed apparent BER given by formula (14), the throughput and rejection probability are different depending on the values of the parameters λ_i and μ_i. If for instance we fix $\Lambda = 1$ (then necessarily $\mu_0 + \mu_1 = 16$ BER) and a very high BER (BER $= 0,01$), then:

• if we imagine $\mu_0 = 0$ and $\mu_1 = 16$ BER (correlated failures), then the throughput is quite high (half the time there is no distortion, half the time there is almost always distortion),

• and if we imagine $\mu_0 = \mu_1$ (uncorrelated failures), then the throughput is almost null.

Now that we have showed that the concept of apparent BER is insufficient, we try to find out the influence of the term m which appears in the formula (14).

In what follows, the number of stations is fixed and equal to $N = 30$. We define

$$\Sigma_m = \{(\Gamma, \Lambda) \in [0,1] \times [0, +\infty[\backslash \frac{\Gamma + \Lambda}{1 + \Lambda} = m\}$$

In order to study the influence of m, we adopt the following method. λ_1 is always fixed ($\lambda_1 = 10^{-5}$). Each time, we fix the apparent BER. With this fixed BER, we examine different values of m (each time we fix m, we directly obtain μ_1). And then, we can move μ_0 and λ_0 in Σ_m to see the influence of the correlation on the throughput and on the rejection probability. In what follows, we will mainly get interested in the evolution of the situation when $m \longrightarrow 1^-$ and $m \longrightarrow 0$. We will refer to check our assumptions to figures 5 and 6, where we have the following curves. We have taken BER$= 5 \cdot 10^{-5}$: the solid curves correspond to the RTS/CTS mechanism, the dashed curves to the BAM, and the dotted curves are the curves obtained from simulation to check the case $m = 0.95$ (in this case, our analytical results could be not as precise as they were when m is low because the approximation made in III-A should not hold anymore). So finally, curves (a) and (a') are obtained for $m = 0.95$ under the RTS mechanism and the BAM respectively, (b) and (b') for $m = 0.143$ under the RTS mechanism and the BAM respectively, and (c) and (c') for $m = 0.05$ under the RTS mechanism and the BAM respectively.

We have $m = 1 \iff \Gamma = 1$, so uncorrelation is equivalent to $m = 1$, and we consequently suppose that the system evolves to the uncorrelated situation for $m \longrightarrow 1^-$. The point is that for $m \approx 1$, we necessarily have $\mu_1 \approx$ BER, but we can still have $\mu_0 = 0$, i.e. even if m is very close to 1, the situation can be correlated. Hence, the evolution of the system is not obvious. Still figures 5 and 6 show that for high value of m (curves (a) and (a'), case $m = 0.95$), both the throughput and the rejection probability stay nearly the same: so as we

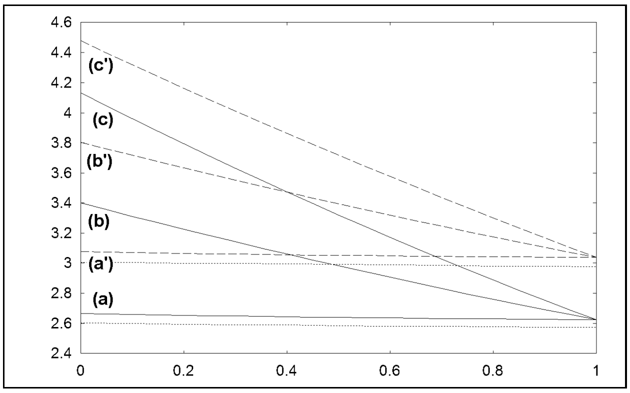

Figure 5. Influence of m: throughput vs. $\frac{\mu_0}{\text{BER}}$ for different values of m

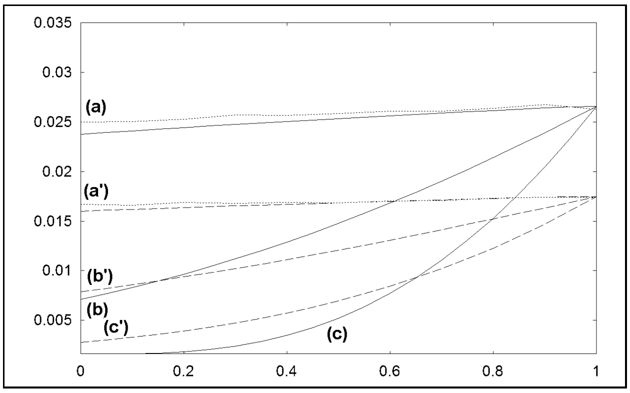

Figure 6. Influence of m: rejection probability vs. $\frac{\mu_0}{\text{BER}}$ for different values of m

thought it, the situation evolves continuously to the uncorrelated one. This can be understood as follows. Let us divide the range of acceptable values for Γ in two intervals:

• in the first interval, Γ is close to m. As m is high (i.e. close to 1), Γ is close to 1 too, and the situation is then obviously close to the uncorrelated situation

• in the second interval, Γ is not close enough to m for us to say that the μ_i are close to each other. But in this case, the slope of the line supporting Σ_m being very low, Λ is already very high. It means that λ_0 is very high, and that the channel is almost always in the bad state, in which μ_1 is set.

On the other side, $m \longrightarrow 0 \iff \mu_1 \longrightarrow +\infty$, and at the limit $\Gamma = \Lambda = 0$. $\Lambda = 0$ means that either $\lambda_0 = 0$, either $\lambda_1 = +\infty$. In both cases, the channel is always in the good state, so the influence of correlation is crucial. Indeed, for given BER and m, μ_0 can always vary between 0 and BER, and the channel is always in the state 0. So the channel is most of the time in a state in which the noise can be either very high, or null.

Finally, we have checked here the very natural idea according to which the correlation is very sensible (for a given apparent BER) when the BER in the bad state is high.

To conclude, let us note an interesting point. Figure 5 shows that the throughput is always better under the BAM than under the RTS/CTS mechanism. So concerning the throughput, in our given case, the correlation does not seem to have influence on $\overline{P}_{opt}$. On the other hand, concerning the rejection probability, we see that when m is low (on figure 6 for curves (b) and (c), case $m \approx 0.143$ and $m = 0.05$), the RTS/CTS mechanism is better and provides a lower rejection probability than the BAM, until a certain threshold. So here we see that the correlation has a very sensible role.

VII. Conclusions

In this paper, a continuation of [1], [3] and [6]–[9], we have developed an analytical method for estimating the throughput of a Wi-Fi ad hoc LAN operating under saturation and in the presence of noise. Besides the throughput, our method allows evaluating the probability of a packet rejection due to attaining the retry number threshold [12]. Moreover, this analytical method, in continuation with [7] for Wi-Fi network allows performance evaluation in case of correlated failures inherent to realistic wireless channels. The failures correlation has been described with the modified two-states Gilbert model [4], where sojourn times in each of channel states are assumed to be exponentially distributed.

According to numerical results obtained by both the developed method and simulation, our method is quite exact: the errors never exceed 5% with throughput estimation and 2% with rejection probability estimation. This method provides a high speed of calculating the values of performance indices, which has allowed us to perform the exhaustive search of optimal RTS threshold and to show how the RTS/CTS efficiency depends on failures correlation.

As a future research activity, we propose extensions of this method to take into account a possible presence of hidden stations, which would particularly interesting as far as the RTS/CTS mechanism is meant too to deal efficiently with this problem, as well as to consider and to optimize a channel rate switching mechanism what promises to be effective in the case of correlated failures.

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